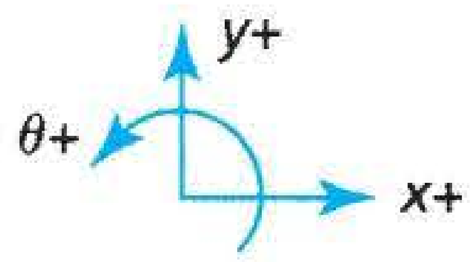
CENGAGE

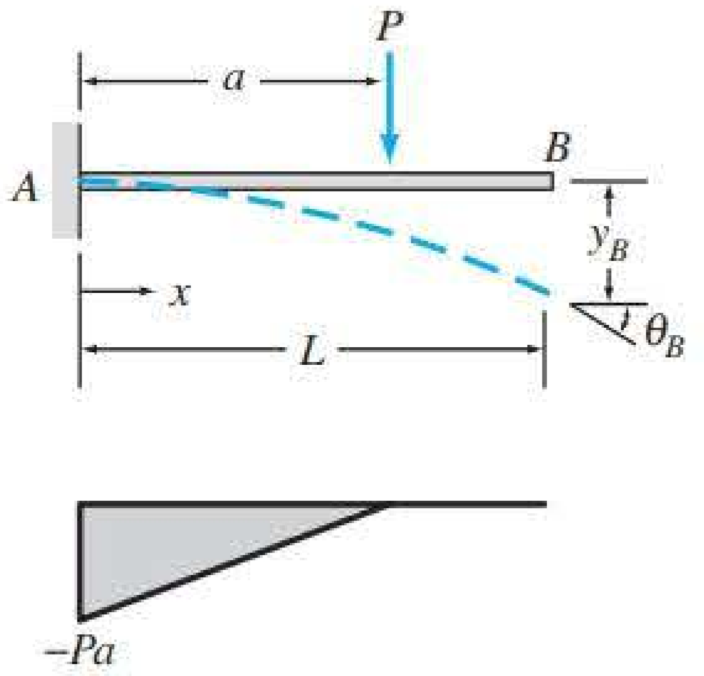
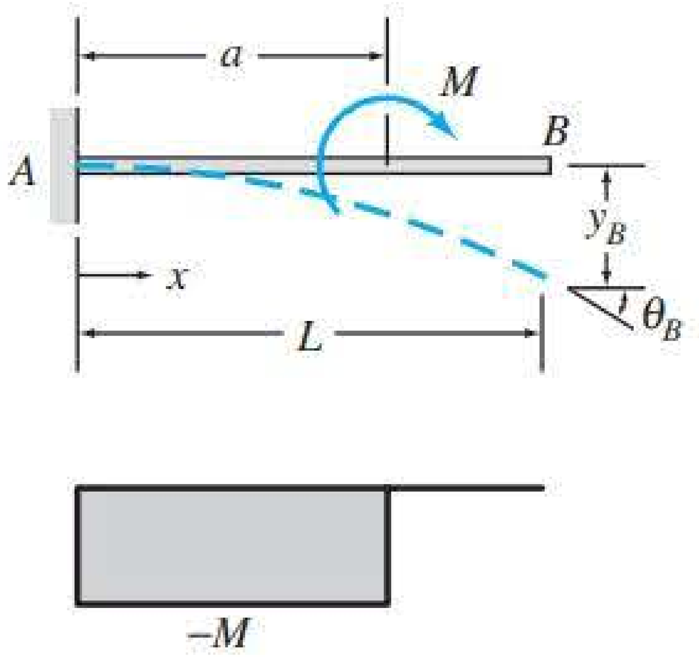
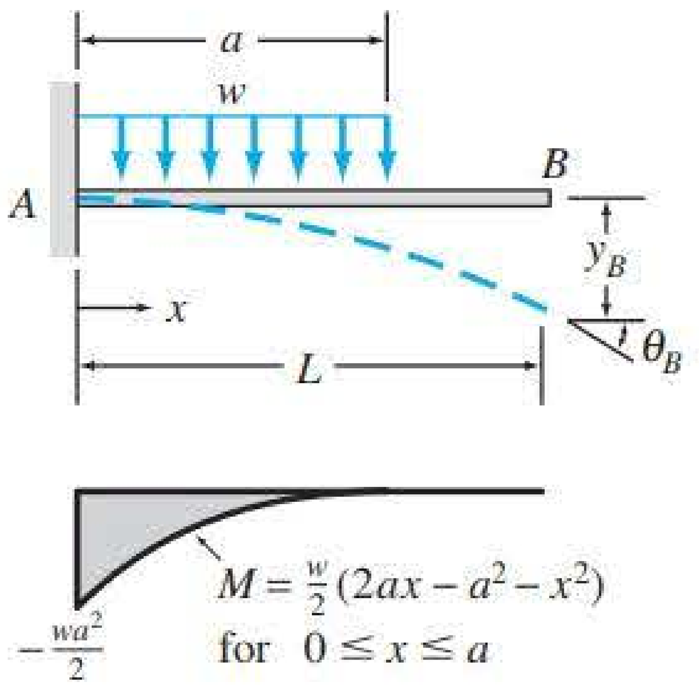
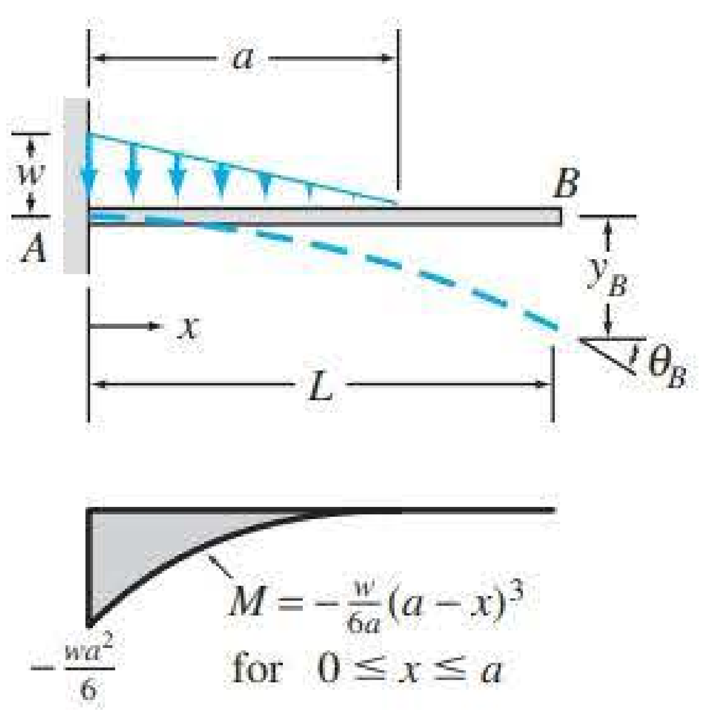
STRUCTURAL Analysis

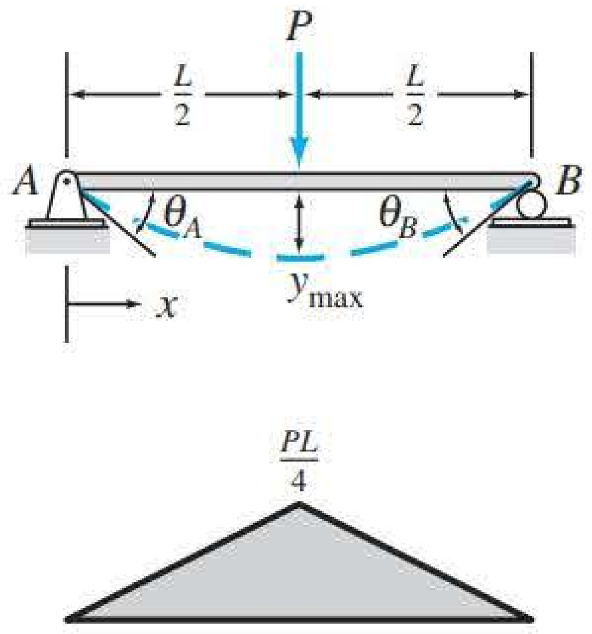
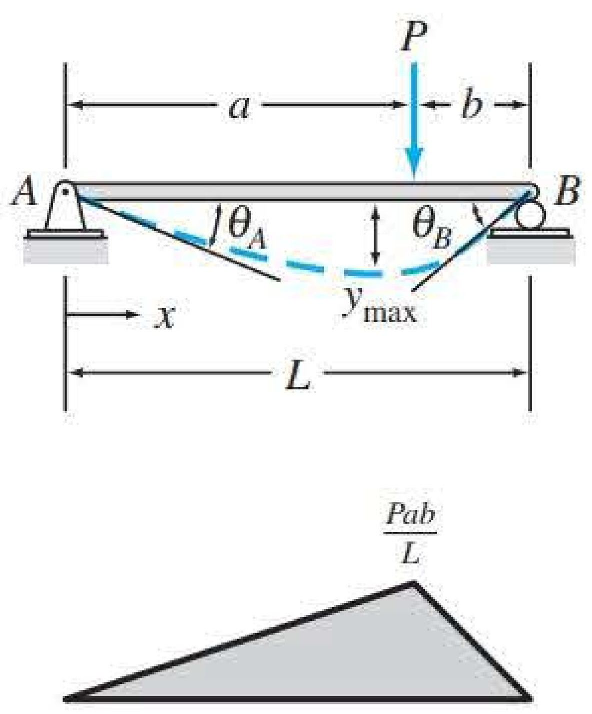
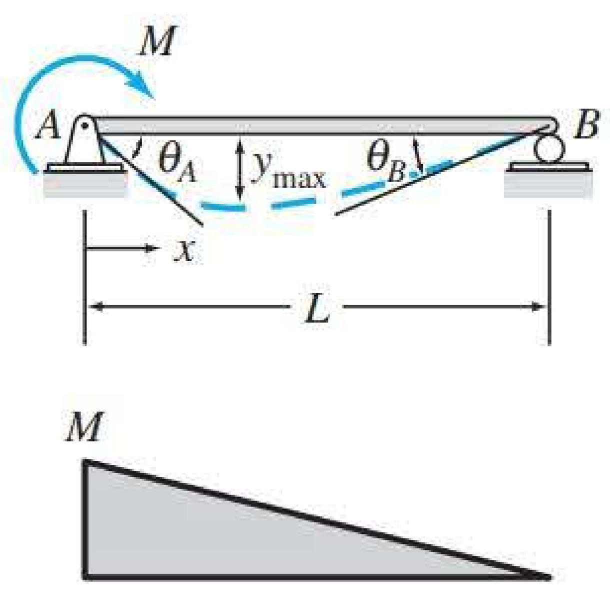
SIXTH EDITION

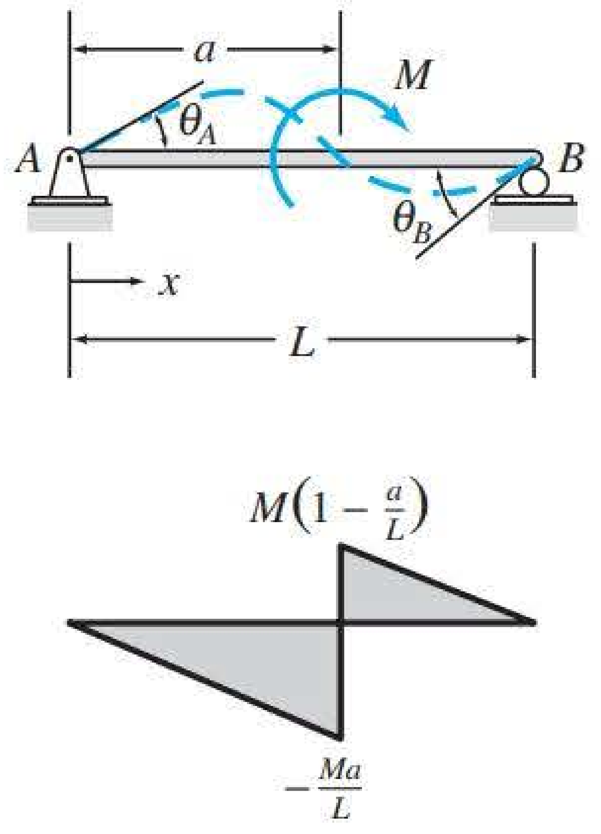
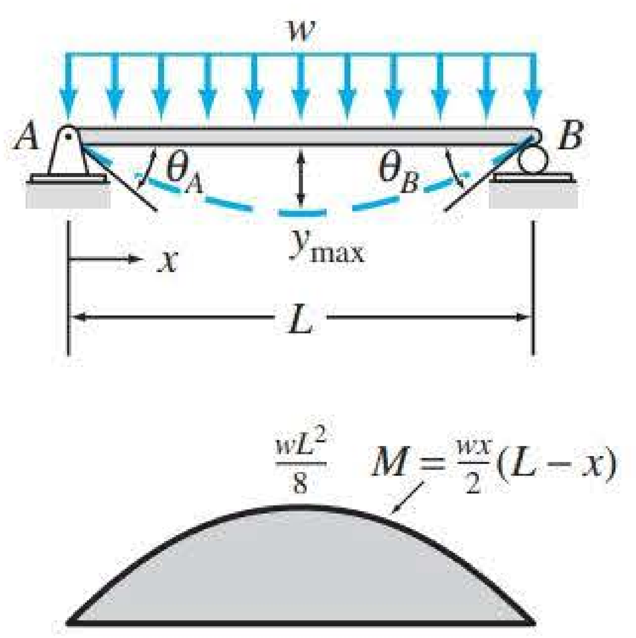
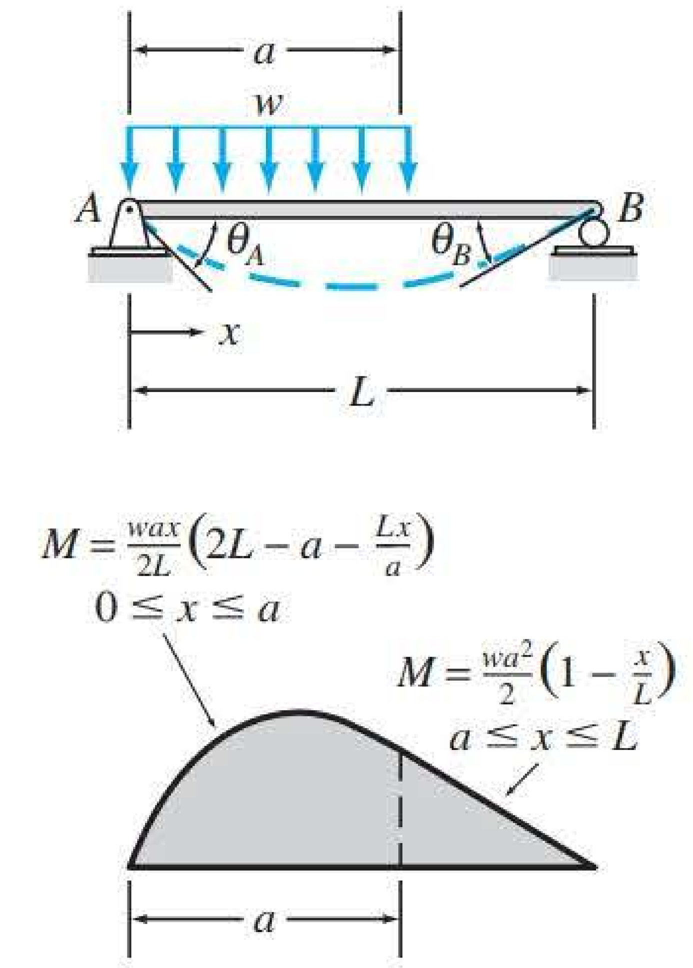
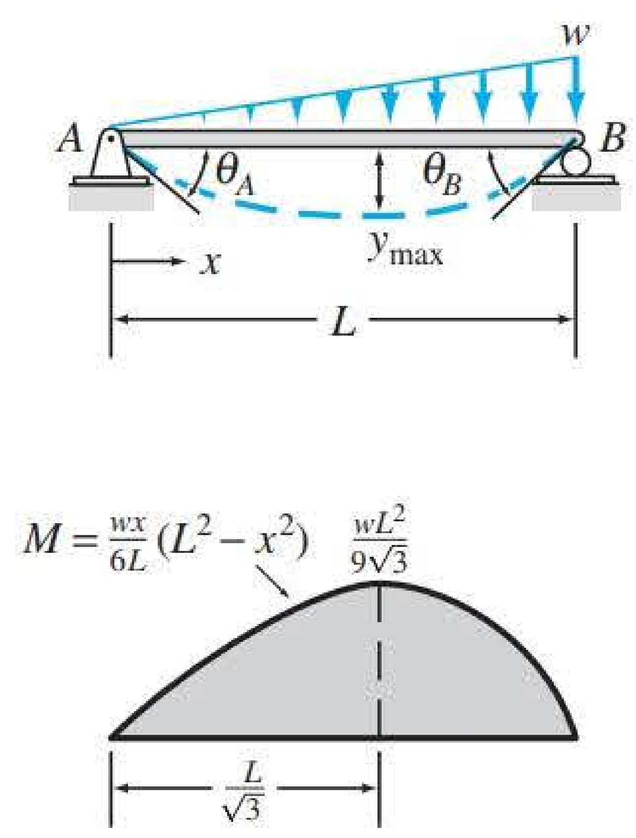
ASLAM KASSIMALI

BENDING MOMENTS, SLOPES, AND DEFLECTIONS OF PRISMATIC BEAMS UNDER VARIOUS LOADING CONDITIONS



<table>
<tr><th>Beam, Loading, and Bending Moment Diagram</th><th>Equations for Slope and Deflection</th></tr>
<tr><td>
</td><td>$0 \le x \le a$:
$\theta = \frac{P}{2EI}(x^2 - 2ax)$
$y = \frac{P}{6EI}(x^3 - 3ax^2)$
$a \le x \le L$:
$\theta = -\frac{Pa^2}{2EI}$
$y = \frac{Pa^2}{6EI}(a - 3x)$
$\theta_B = -\frac{Pa^2}{2EI}$; $y_B = -\frac{Pa^2}{6EI}(3L - a)$</td></tr>
<tr><td>
</td><td>$0 \le x \le a$:
$\theta = -\frac{Mx}{EI}$
$y = -\frac{Mx^2}{2EI}$
$a \le x \le L$:
$\theta = -\frac{Ma}{EI}$
$y = \frac{Ma}{2EI}(a - 2x)$
$\theta_B = -\frac{Ma}{EI}$; $y_B = -\frac{Ma}{2EI}(2L - a)$</td></tr>
<tr><td>
</td><td>$0 \le x \le a$:
$\theta = \frac{w}{6EI}(3ax^2 - 3a^2x - x^3)$
$y = \frac{w}{24EI}(4ax^3 - 6a^2x^2 - x^4)$
$a \le x \le L$:
$\theta = -\frac{wa^3}{6EI}$
$y = \frac{wa^3}{24EI}(a - 4x)$
$\theta_B = -\frac{wa^3}{6EI}$; $y_B = -\frac{wa^3}{24EI}(4L - a)$</td></tr>
<tr><td>
</td><td>$0 \le x \le a$:
$\theta = \frac{w}{24EIa}(x^4 - 4ax^3 + 6a^2x^2 - 4a^3x)$
$y = \frac{w}{120EIa}(x^5 - 5ax^4 + 10a^2x^3 - 10a^3x^2)$
$a \le x \le L$:
$\theta = -\frac{wa^3}{24EI}$
$y = \frac{wa^3}{120EI}(-5x + a)$
$\theta_B = -\frac{wa^3}{24EI}$; $y_B = -\frac{wa^3}{120EI}(5L - a)$</td></tr>
</table>

Beam, Loading, and Bending Moment Diagram	Equations for Slope and Deflection
	$0 \le x \le \frac{L}{2}$: $\theta = \frac{P}{16EI}(4x^2 - L^2)$ $y = \frac{P}{48EI}(4x^3 - 3L^2x)$ $\theta_A = -\frac{PL^2}{16EI};\ \ \theta_B = \frac{PL^2}{16EI}$ $y_{\max} = -\frac{PL^3}{48EI}$
	$0 \le x \le a$: $\theta = \frac{Pb}{6EIL}(3x^2 + b^2 - L^2)$ $y = \frac{Pb}{6EIL}(x^3 + b^2x - L^2x)$ $a \le x \le L$: $\theta = \frac{Pa}{6EIL}[L^2 - a^2 - 3(L - x)^2]$ $y = \frac{Pa(L-x)}{6EIL}(x^2 + a^2 - 2Lx)$ $\theta_A = -\frac{Pb}{6EIL}(L^2 - b^2)$ $\theta_B = \frac{Pa}{6EIL}(L^2 - a^2)$ For $a \ge b$: $y_{\max} = -\frac{Pb}{9\sqrt{3}EIL}(L^2 - b^2)^{3/2}$ at $x = \left(\frac{L^2 - b^2}{3}\right)^{1/2}$
	$\theta = -\frac{M}{6EIL}(3x^2 - 6Lx + 2L^2)$ $y = -\frac{M}{6EIL}(x^3 - 3Lx^2 + 2L^2x)$ $\theta_A = -\frac{ML}{3EI};\ \ \theta_B = \frac{ML}{6EI}$ $y_{\max} = -\frac{ML^2}{9\sqrt{3}EI}$ at $x = L\left(1 - \frac{1}{\sqrt{3}}\right)$

Beam, Loading, and Bending Moment Diagram	Equations for Slope and Deflection
	$0 \le x \le a$: $\theta = \frac{M}{6EIL}(-3x^2 + 6aL - 3a^2 - 2L^2)$ $y = \frac{M}{6EIL}(-x^3 + 6aLx - 3a^2x - 2L^2x)$ $\theta_A = \frac{M}{6EIL}(6aL - 3a^2 - 2L^2)$ $\theta_B = \frac{M}{6EIL}(L^2 - 3a^2)$
	$\theta = -\frac{w}{24EI}(4x^3 - 6Lx^2 + L^3)$ $y = -\frac{w}{24EI}(x^4 - 2Lx^3 + L^3x)$ $\theta_A = -\frac{wL^3}{24EI}$ $\theta_B = \frac{wL^3}{24EI}$ $y_{max} = -\frac{5wL^4}{384EI}$ at $x = \frac{L}{2}$
	$0 \le x \le a$: $\theta = -\frac{w}{24EIL}[4Lx^3 - 6a(2L-a)x^2 + a^2(2L-a)^2]$ $y = -\frac{w}{24EIL}[Lx^4 - 2a(2L-a)x^3 + a^2(2L-a)^2x]$ $a \le x \le L$: $\theta = -\frac{wa^2}{24EIL}(6x^2 - 12Lx + a^2 + 4L^2)$ $y = -\frac{wa^2}{24EIL}(L-x)(-2x^2 + 4Lx - a^2)$ $\theta_A = -\frac{wa^2}{24EIL}(2L-a)^2$ $\theta_B = \frac{wa^2}{24EIL}(2L^2 - a^2)$
	$\theta = -\frac{w}{360EIL}(15x^4 - 30L^2x^2 + 7L^4)$ $y = -\frac{w}{360EIL}(3x^5 - 10L^2x^3 + 7L^4x)$ $\theta_A = -\frac{7wL^3}{360EI}$ $\theta_B = \frac{wL^3}{45EI}$ $y_{max} = -0.00652\frac{wL^4}{EI}$ at $x = 0.5193L$

Structural Analysis

Sixth Edition

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Aslam Kassimali
Southern Illinois University-Carbondale

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Aslam Kassimali

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IN MEMORY OF *AMI* AND *APAJAN*

Contents

Preface

The objective of this book is to develop an understanding of the basic principles of structural analysis. Emphasizing the intuitive classical approach, *Structural Analysis* covers the analysis of statically determinate and indeterminate beams, trusses, and rigid frames. It also presents an introduction to the matrix analysis of structures.

The book is divided into three parts. Part One presents a general introduction to the subject of structural engineering. It includes a chapter devoted entirely to the topic of loads because attention to this important topic is generally lacking in many civil engineering curricula. Part Two, consisting of Chapters 3 through 10, covers the analysis of statically determinate beams, trusses, and rigid frames. The chapters on deflections (Chapters 6 and 7) are placed before those on influence lines (Chapters 8 and 9) so that influence lines for deflections can be included in the latter chapters. This part also contains a chapter on the analysis of symmetric structures (Chapter 10). Part Three of the book, Chapters 11 through 17, covers the analysis of statically indeterminate structures. The format of the book is flexible to enable instructors to emphasize topics that are consistent with the goals of the course.

Each chapter of the book begins with an introductory section defining its objective and ends with a summary section outlining its salient features. An important general feature of the book is the inclusion of step-by-step procedures for analysis to enable students to make an easier transition from theory to problem solving. Numerous solved examples are provided to illustrate the application of the fundamental concepts.

A computer program for analyzing plane framed structures is available on the publisher's website at https://login.cengage.com. It is also available at https://www.cengage.com/engineering/kassimali/software. This interactive software can be used to simulate a variety of structural and loading configurations and to determine cause versus effect relationships between loading and various structural parameters, thereby enhancing the students' understanding of the behavior of structures. The software shows deflected shapes of structures to enhance students' understanding of structural response due to various types of loadings. It can also include the effects of support settlements, temperature changes, and fabrication errors in the analysis. A solutions manual, containing complete solutions to over 600 text exercises, is also available for the instructor.

New to the Sixth Edition

Building upon the original theme of this book, which is that detailed explanations of concepts provide the most effective means of teaching structural analysis, the following improvements and changes have been made in this sixth edition:

- Over 20 percent of the problems from the previous edition have been replaced with new ones.
- The chapter on loads has been revised to meet the provisions of the ASCE/SEI 7-16 Standard and the latest AASHTO-LRFD Specifications.
- The content of the chapter on the application of influence lines has been updated to incorporate the current HL-93 truck/tandem loadings as per AASHTO-LRFD Specifications.
- Throughout the book, there are numerous other revisions to enhance clarity and reinforce concepts. These include several new and upgraded examples in Chapters 3, 5, 10, 13, 15, 16 and in Appendix C, as well as an expanded discussion of the analysis of plane frames via classical versus matrix methods (Chapter 17).
- Some photographs have been replaced with new ones, and the page layout has been redesigned to enhance clarity.
- Finally, the computer software has been upgraded and recompiled to make it compatible with the latest versions of Microsoft Windows.

Ancillaries for the Sixth Edition

Worked-out solutions to all end-of-chapter problems are provided in the Instructors Solutions Manual and are available digitally to registered instructors on the instructor resources web site. Image Banks containing every figure in the book are also available at https://login.cengage.com. The computer program for analyzing plane framed structures is available for both students and instructors through either https://login.cengage.com or https://www.cengage.com/engineering/kassimali/software.

Acknowledgments

I wish to express my thanks to Timothy Anderson, Mona Zeftel, and Alexander Sham of Cengage Learning for their constant support and encouragement throughout this project, and to Rose Kernan for all her help during the

production phase. The comments and suggestions for improvement from colleagues and students who have used previous editions are gratefully acknowledged. All of their suggestions were carefully considered, and implemented whenever possible. Thanks are due to the following reviewers for their careful reviews of the manuscripts of the various editions, and for their constructive suggestions:

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University of Dayton

Finally, I would like to express my loving appreciation to my wife, Maureen, for her constant encouragement and help in preparing this manuscript, and to my sons, Jamil and Nadim, for their love, understanding, and patience.

Aslam Kassimali

About the Author

Aslam Kassimali was born in Karachi, Pakistan. He received his Bachelor of Engineering (B.E.) degree in civil engineering from the University of Karachi (N.E.D. College) in Pakistan in 1969. In 1971, he earned a Master of Engineering (M.E.) degree in civil engineering from Iowa State University in Ames, Iowa, USA. After completing further studies and research at the University of Missouri at Columbia in the USA, he received Master of Science (M.S.) and Ph.D. degrees in civil engineering in 1974 and 1976, respectively.

His practical experience includes work as a Structural Design Engineer for Lutz, Daily and Brain, Consulting Engineers, Shawnee Mission, Kansas (USA), from January to July 1973, and as a Structural Engineering Specialist and Analyst for Sargent & Lundy Engineers in Chicago, Illinois (USA) from 1978 to 1980. He joined Southern Illinois University—Carbondale (USA) as an Assistant Professor in 1980, and was promoted to the rank of Professor in 1993. Consistently recognized for teaching excellence, Dr. Kassimali has received over 20 awards for outstanding teaching at Southern Illinois University—Carbondale, and was awarded the title of Distinguished Teacher in 2004. He is currently a Professor and Distinguished Teacher in the Department of Civil & Environmental Engineering at Southern Illinois University in Carbondale, Illinois (USA). He has authored and co-authored four textbooks on structural analysis and mechanics, and has published a number of papers in the area of nonlinear structural analysis.

Dr. Kassimali is a life member of the American Society of Civil Engineers (ASCE) and has served on the ASCE Structural Division Committees on *Shock and Vibratory Effects, Special Structures,* and *Methods of Analysis*.

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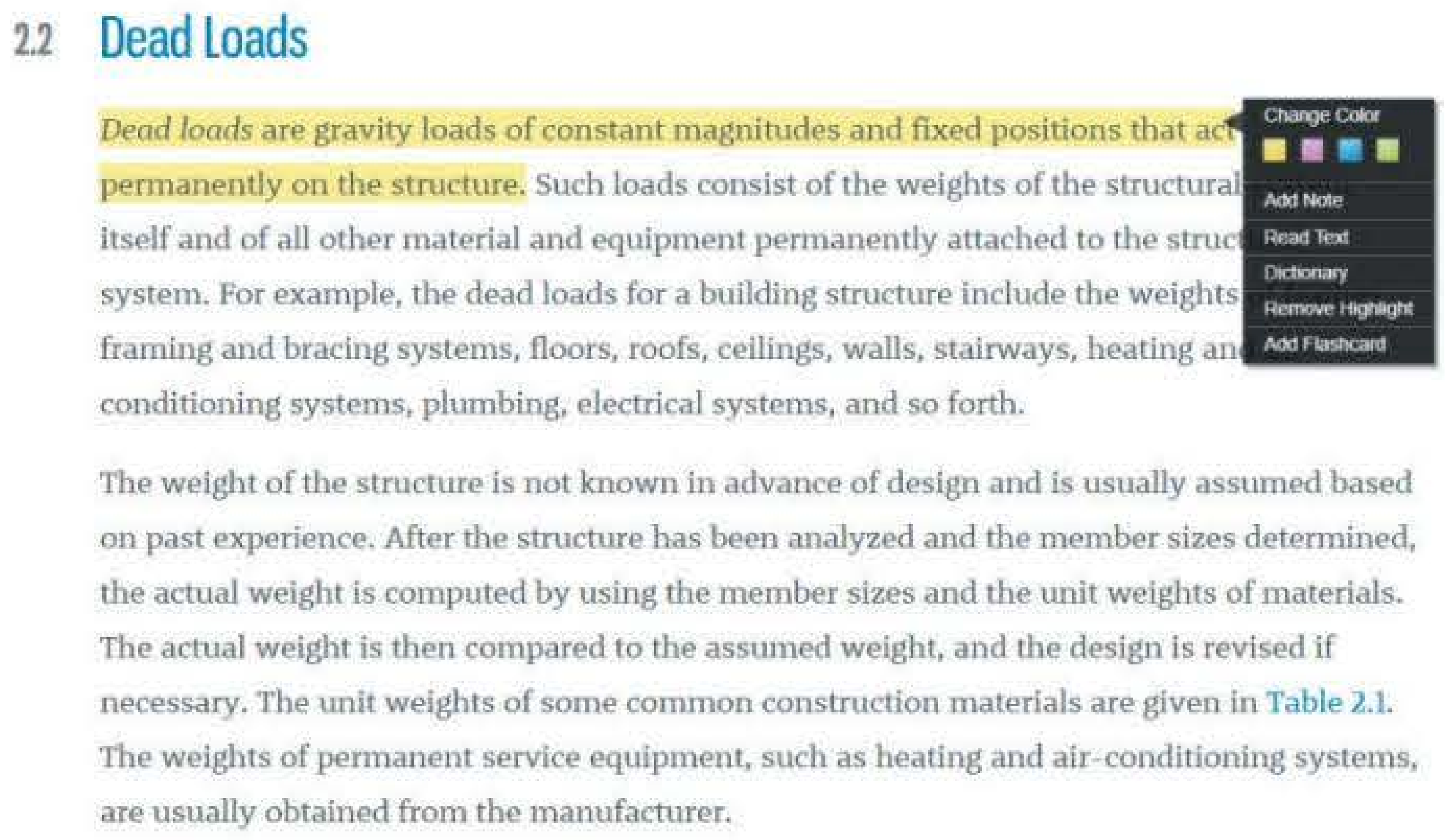

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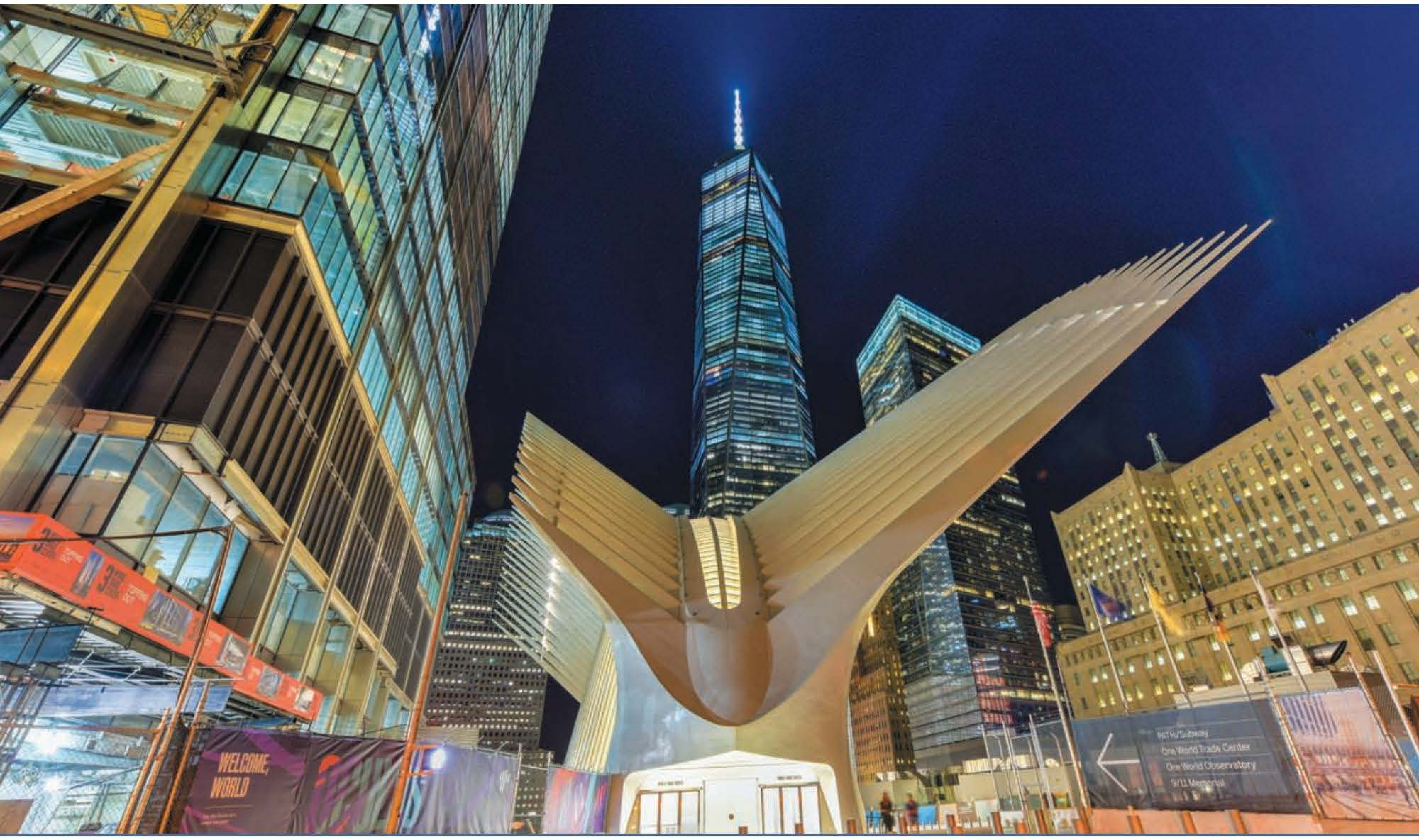

Part One

Introduction to Structural Analysis and Loads

1

Introduction to Structural Analysis

Marina City District, Chicago

Hisham Ibrahim/Photographer's Choice RF/Getty Images

Structural analysis is the prediction of the performance of a given structure under prescribed loads and/or other external effects, such as support movements and temperature changes. The performance characteristics commonly of interest in the design of structures are (1) stresses or stress resultants, such as axial forces, shear forces, and bending moments; (2) deflections; and (3) support reactions. Thus, the analysis of a structure usually involves determination of these quantities as caused by a given loading condition. The objective of this text is to present the methods for the analysis of structures in static equilibrium.

This chapter provides a general introduction to the subject of structural analysis. We first give a brief historical background, including names of people whose work is important in the field. Then we discuss the role of structural analysis in structural engineering projects. We describe the five common types of structures: tension and compression structures, trusses, and shear and bending structures. Finally, we consider the development of the simplified models of real structures for the purpose of analysis.

1.1 Historical Background

Since the dawn of history, structural engineering has been an essential part of human endeavor. However, it was not until about the middle of the seventeenth century that engineers began applying the knowledge of mechanics (mathematics and science) in designing structures. Earlier engineering structures were designed by trial and error and by using rules of thumb based on past experience. The fact that some of the magnificent structures from earlier eras, such as Egyptian pyramids (about 3000 B.C.), Greek temples (500–200 B.C.), Roman coliseums and aqueducts (200 B.C.–A.D. 200), and Gothic cathedrals (A.D. 1000–1500), still stand today is a testimonial to the ingenuity of their builders (Fig. 1.1).

Galileo Galilei (1564–1642) is generally considered to be the originator of the theory of structures. In his book entitled *Two New Sciences*, which was published in 1638, Galileo analyzed the failure of some simple structures, including cantilever beams. Although Galileo's predictions of strengths of beams were only approximate, his work laid the foundation for future developments in the theory of structures and ushered in a new era of structural engineering, in which the analytical principles of mechanics and strength of materials would have a major influence on the design of structures.

Following Galileo's pioneering work, the knowledge of structural mechanics advanced at a rapid pace in the second half of the seventeenth century and into the eighteenth century. Among the notable investigators of that period were Robert Hooke (1635–1703), who developed the law of linear relationships between the force and deformation of materials (Hooke's law); Sir Isaac Newton (1642–1727), who formulated the laws of motion and developed calculus; John Bernoulli (1667–1748), who formulated the principle of virtual work; Leonhard Euler (1707–1783), who developed the theory of

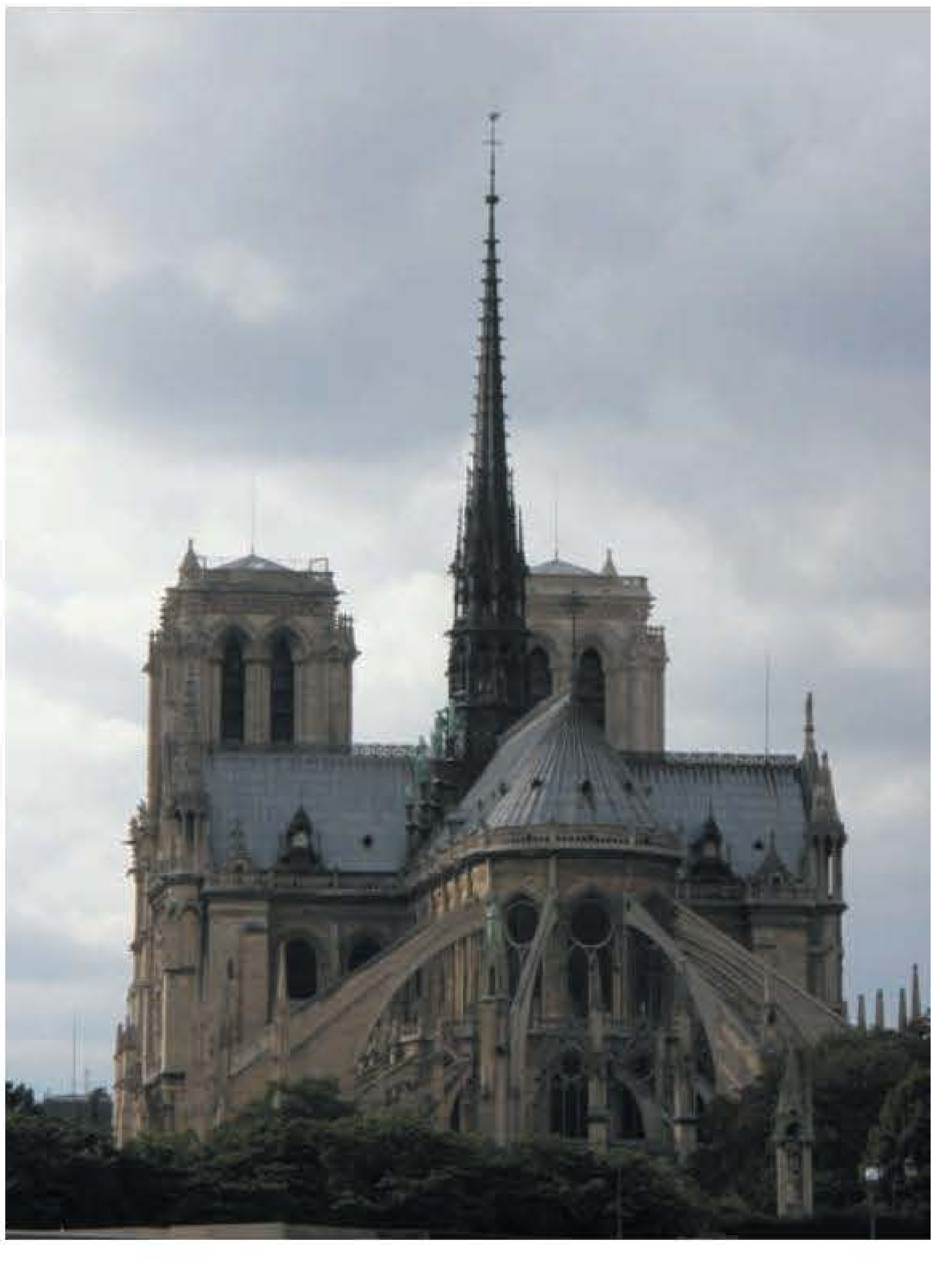

FIG. 1.1 The Cathedral of Notre Dame in Paris Was Completed in the Thirteenth Century
Ritu Manoj Jethani/Shutterstock.com

buckling of columns; and C. A. de Coulomb (1736–1806), who presented the analysis of bending of elastic beams.

In 1826 L. M. Navier (1785–1836) published a treatise on elastic behavior of structures, which is considered to be the first textbook on the modern theory of strength of materials. The development of structural mechanics continued at a tremendous pace throughout the rest of the nineteenth century and into the first half of the twentieth, when most of the classical methods for the analysis of structures described in this text were developed. The important contributors of this period included B. P. Clapeyron (1799–1864), who formulated the three-moment equation for the analysis of continuous beams; J. C. Maxwell (1831–1879), who presented the method of consistent deformations and the law of reciprocal deflections; Otto Mohr (1835–1918), who developed the conjugate-beam method for calculation of deflections and Mohr's circles of stress and strain; Alberto Castigliano (1847–1884), who formulated the theorem of least work; C. E. Greene (1842–1903), who developed the moment-area method; H. Müller-Breslau (1851–1925), who presented a principle for constructing influence lines; G. A. Maney (1888–1947), who developed the slope-deflection method, which is considered to be the precursor of the matrix stiffness method; and Hardy Cross (1885–1959), who developed the moment-distribution method in 1924. The moment-distribution method provided engineers with a simple iterative procedure for analyzing highly statically indeterminate structures. This method, which was the most widely used by structural engineers during the period from about 1930 to 1970, contributed significantly to their understanding of the behavior of statically indeterminate frames. Many structures designed during that period, such as high-rise buildings, would not have been possible without the availability of the moment-distribution method.

The availability of computers in the 1950s revolutionized structural analysis. Because the computer could solve large systems of simultaneous equations, analyses that took days and sometimes weeks in the precomputer era could now be performed in seconds. The development of the current computer-oriented methods of structural analysis can be attributed to, among others, J. H. Argyris, R. W. Clough, S. Kelsey, R. K. Livesley, H. C. Martin, M. T. Turner, E. L. Wilson, and O. C. Zienkiewicz.

1.2 Role of Structural Analysis in Structural Engineering Projects

Structural engineering is the science and art of planning, designing, and constructing safe and economical structures that will serve their intended purposes. Structural analysis is an integral part of any structural engineering project, its function being the prediction of the performance of the proposed structure. A flowchart showing the various phases of a typical structural engineering project is presented in Fig. 1.2. As this diagram indicates, the process is an iterative one, and it generally consists of the following steps:

1. ***Planning Phase*** The planning phase usually involves the establishment of the functional requirements of the proposed structure, the general layout and dimensions of the structure, and consideration of the possible types of structures (e.g., rigid frame or truss) that may be feasible and the types of materials to be used (e.g., structural steel

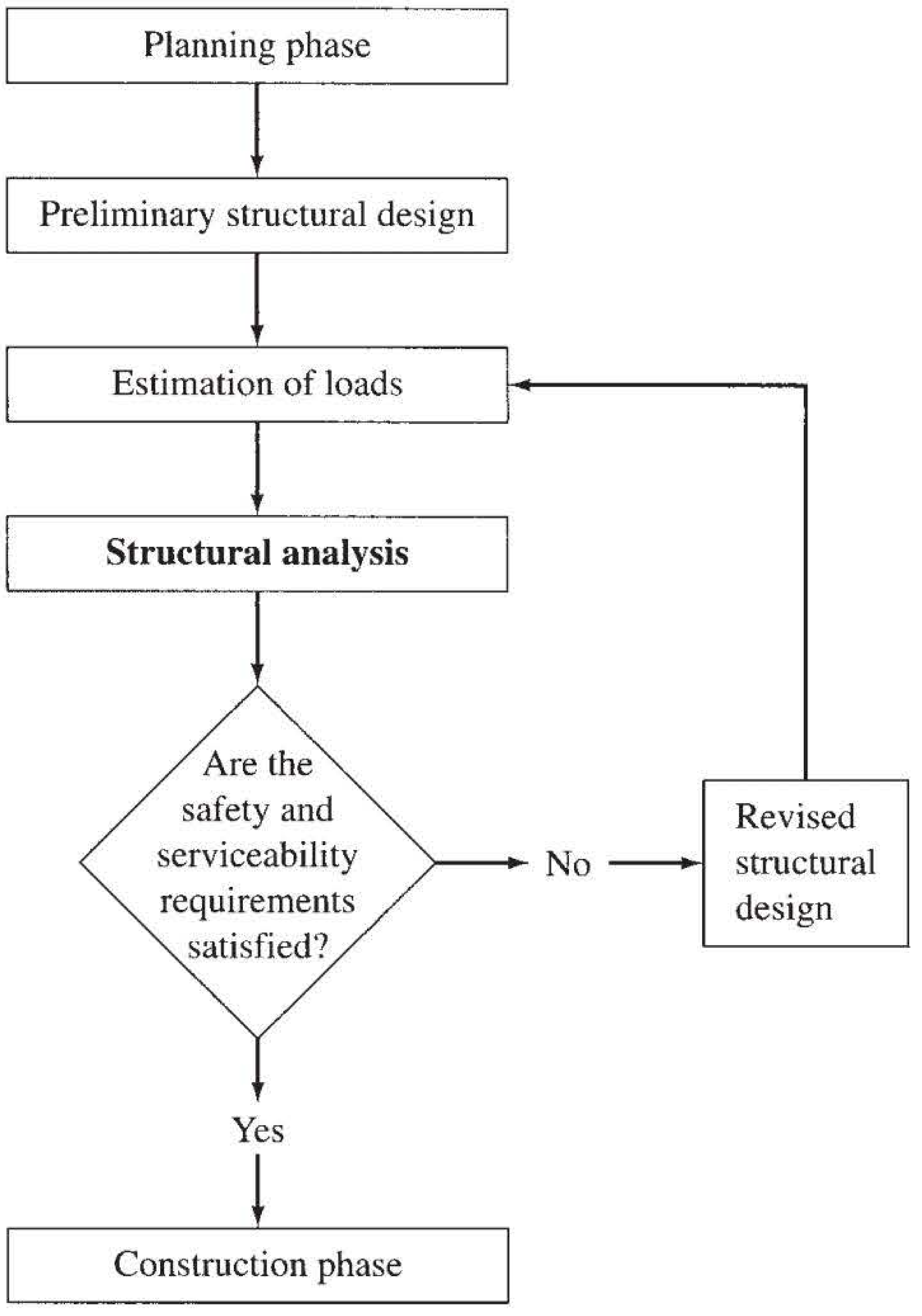

FIG. 1.2 Phases of a Typical Structural Engineering Project

or reinforced concrete). This phase may also involve consideration of nonstructural factors, such as aesthetics, environmental impact of the structure, and so on. The outcome of this phase is usually a structural system that meets the functional requirements and is expected to be the most economical. This phase is perhaps the most crucial one of the entire project and requires experience and knowledge of construction practices in addition to a thorough understanding of the behavior of structures.

2. ***Preliminary Structural Design*** In the preliminary structural design phase, the sizes of the various members of the structural system selected in the planning phase are estimated based on approximate analysis, past experience, and code requirements. The member sizes thus selected are used in the next phase to estimate the weight of the structure.
3. ***Estimation of Loads*** Estimation of loads involves determination of all the loads that can be expected to act on the structure.
4. ***Structural Analysis*** In structural analysis, the values of the loads are used to carry out an analysis of the structure in order to determine the stresses or stress resultants in the members and the deflections at various points of the structure.
5. ***Safety and Serviceability Checks*** The results of the analysis are used to determine whether or not the structure satisfies the safety

and serviceability requirements of the design codes. If these requirements are satisfied, then the design drawings and the construction specifications are prepared, and the construction phase begins.

6. ***Revised Structural Design*** If the code requirements are not satisfied, then the member sizes are revised, and phases 3 through 5 are repeated until all the safety and serviceability requirements are satisfied.

Except for a discussion of the types of loads that can be expected to act on structures (Chapter 2), our primary focus in this text will be on the analysis of structures.

1.3 Classification of Structures

As discussed in the preceding section, perhaps the most important decision made by a structural engineer in implementing an engineering project is the selection of the type of structure to be used for supporting or transmitting loads. Commonly used structures can be classified into five basic categories, depending on the type of primary stresses that may develop in their members under major design loads. However, it should be realized that any two or more of the basic structural types described in the following may be combined in a single structure, such as a building or a bridge, to meet the structure's functional requirements.

Tension Structures

The members of tension structures are subjected to pure tension under the action of external loads. Because the tensile stress is distributed uniformly over the cross-sectional areas of members, the material of such a structure is utilized in the most efficient manner. Tension structures composed of flexible steel cables are frequently employed to support bridges and long-span roofs. Because of their flexibility, cables have negligible bending stiffness and can develop only tension. Thus, under external loads, a cable adopts a shape that enables it to support the load by tensile forces alone. In other words, the shape of a cable changes as the loads acting on it change. As an example, the shapes that a single cable may assume under two different loading conditions are shown in Fig. 1.3.

Figure 1.4 shows a familiar type of cable structure—the *suspension bridge*. In a suspension bridge, the roadway is suspended from two main cables by means of vertical hangers. The main cables pass over a pair of towers and are anchored into solid rock or a concrete foundation at their ends. Because suspension bridges and other cable structures lack stiffness in lateral directions, they are susceptible to wind-induced oscillations (see Fig. 1.5). Bracing or stiffening systems are therefore provided to reduce such oscillations.

Besides cable structures, other examples of tension structures include vertical rods used as hangers (for example, to support balconies or tanks) and membrane structures such as tents and roofs of large-span domes (Fig. 1.6).

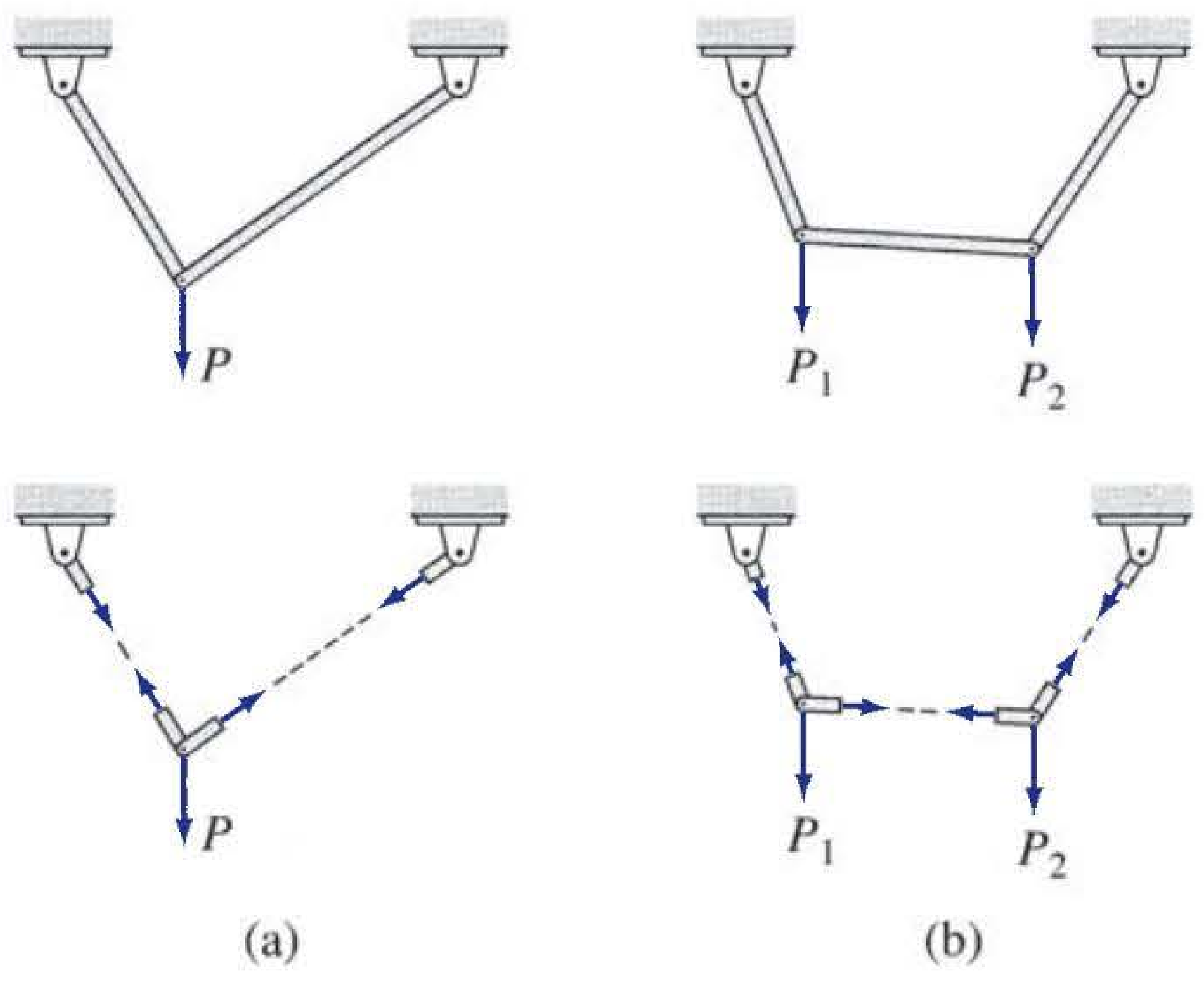

FIG. 1.3

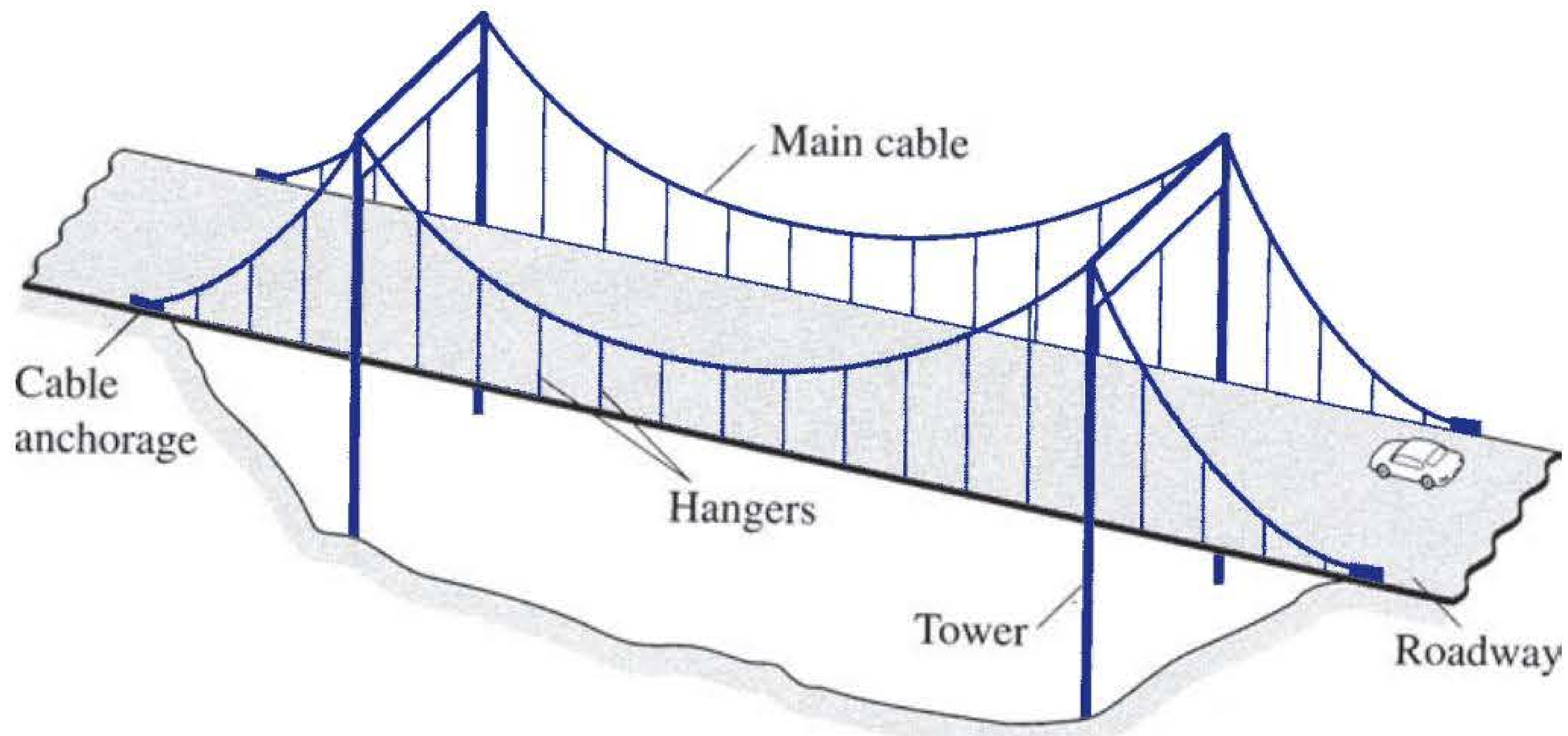

FIG. 1.4 Suspension Bridge

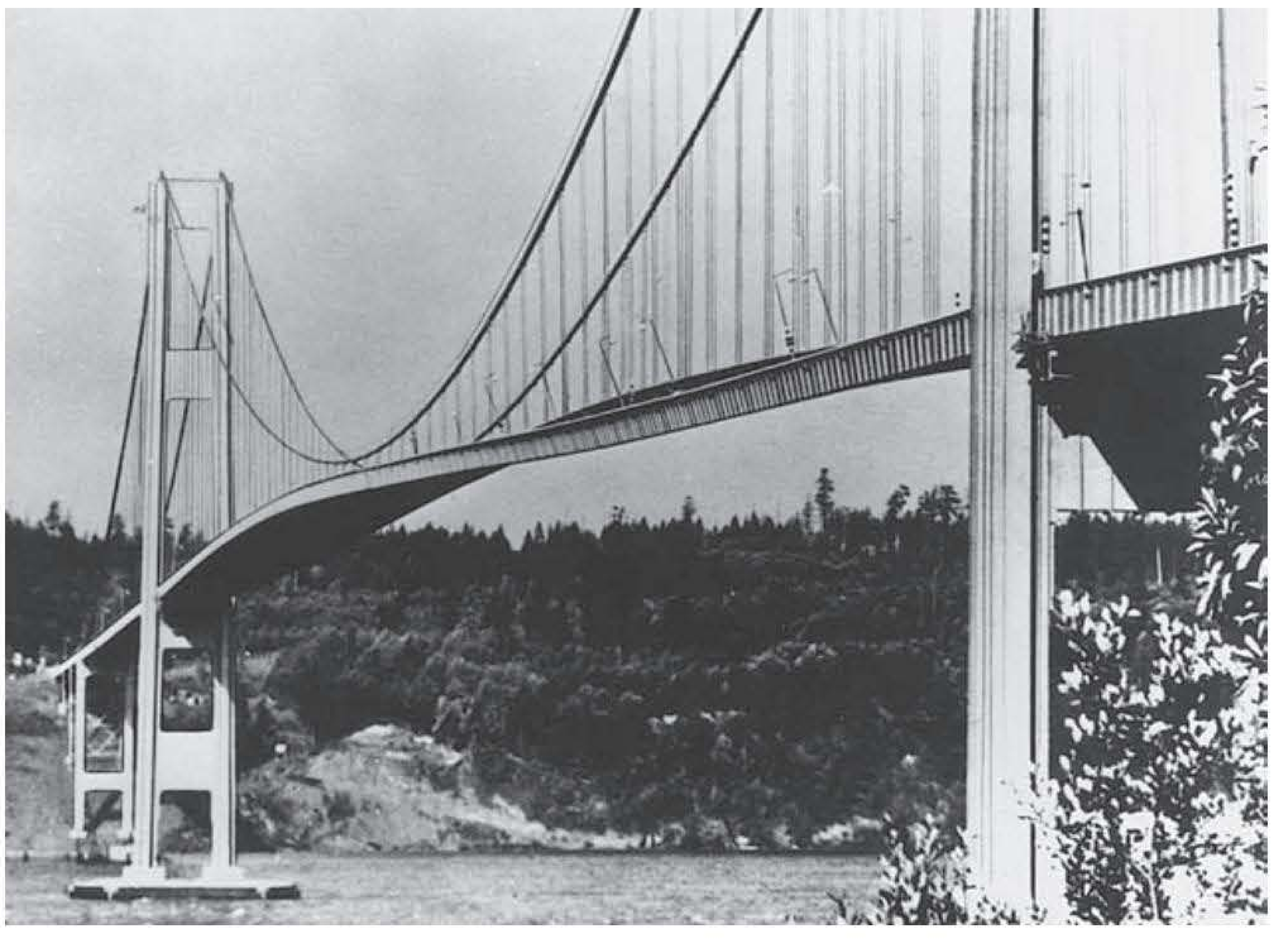

FIG. 1.5 Tacoma Narrows Bridge Oscillating before Its Collapse in 1940
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